Finite difference time domain analysis for measuring transmission and reflection coefficients in negative index materials

Jae-Hong Park and Iam-Choon Khoo
Department of Electrical Engineering, Pennsylvania State University, University Park, PA, 16802

Abstract-We will discuss the reflection and transmission coefficients in the negative index materials. Based on the finite differential time domain approach, we obtain the reflection and transmission coefficients depending on permittivity and permeability at a fixed frequency. The transmissivity and reflectivity will be also examined. Simulation results are in good agreement with the theoretical ones. This technique can be easily extended to the multi-layered structure with negative index materials to design the optical system.

1. INTRODUCTION

A double negative (DNG) material denotes metamaterial having simultaneously negative permittivity and permeability [1]. It has been intensively studied during the past few years because of its scientific interest such as the negative refraction, the reversed Doppler shift and backward waves together with its new practical applications [2]. Recently, it is reported that permittivity and permeability can be tuned over wide ranges from negative even to positive in nano-sphere dispersed liquid crystals [3]. Therefore, for the future applications, the method to obtain the optical properties in DNG materials depending on permittivity and permeability is greatly needed. For this purpose, the finite difference time domain (FDTD) technique [4] can be very powerful tool. Although there has been a lot of discussion theoretically and experimentally, the practical FDTD technique to measure the coefficients as a function of permittivity and permeability at a certain fixed frequency has not been reported in detail yet.

In the paper, we will discuss how to obtain the reflection and transmission coefficient in DNG materials using well established FDTD technique [9,10] and compare the results with the theoretical calculations. Using a normally incident monochromatic plane wave, the electric and magnetic fields depending permittivity and permeability of DNG material adjacent to free space is calculated. The transmitted and reflected fields are recorded at some observation point near the interface between free space and the DNG materials. These recorded fields are converted to the signals in frequency domain using a Fourier transform and then the transmission and reflection coefficients are obtained respectively. We also examine the power flow and measure transmissivity and reflectivity. There are good agreements between the theory and FDTD.

2. TRANSMISSION AND REFLECTION COEFFICIENTS

When a semi-infinite slab of any material having the permittivity ε_t and the permeability μ_t is placed in free space and an incident wave strikes this material from free space at an angle θ_i, we can obtain the transmission and reflection coefficients for each polarized wave as follows.

$$\begin{cases} t_s = \dfrac{2\cos\theta_i}{\cos\theta_i + \cos\theta_t\left(\varepsilon_r/\mu_r\right)^{1/2}} \\ r_s = \dfrac{\cos\theta_i - \cos\theta_t\left(\varepsilon_r/\mu_r\right)^{1/2}}{\cos\theta_i + \cos\theta_t\left(\varepsilon_r/\mu_r\right)^{1/2}} \end{cases} \quad \text{for s-wave} \qquad (1)$$

$$\begin{cases} t_p = \dfrac{2\cos\theta_i}{\cos\theta_i\left(\varepsilon_r/\mu_r\right)^{1/2} + \cos\theta_t} \\ r_p = \dfrac{\cos\theta_i\left(\varepsilon_r/\mu_r\right)^{1/2} - \cos\theta_t}{\cos\theta_i\left(\varepsilon_r/\mu_r\right)^{1/2} + \cos\theta_t} \end{cases} \quad \text{for p-wave} \qquad (2)$$

where ε_r and μ_r are the relative permittivity and permeability of the DNG material. In case of the normal incidence, the amplitude of each coefficient is independent of the polarization. Note that in the DNG material with $\varepsilon_r < 0$ and $\mu_r < 0$, the refractive index has negative value as $n = -\left(\varepsilon_r\mu_r\right)^{1/2}$ but the transmission and reflection coefficients are the same with those of the material with $\varepsilon_r > 0$ and $\mu_r > 0$.

3. FDTD SIMULATION MODEL

If we assume E_y and H_x as the 1-d field components and use $\exp(-j\omega t)$ time convention in the frequency domain, the Maxwell's equations can be expressed as

$$\begin{aligned} j\omega B_x &= -\frac{\partial E_y}{\partial z},\ H_x = \mu(\omega)^{-1} B_x, \\ j\omega D_y &= -\frac{\partial H_x}{\partial z},\ E_y = \varepsilon(\omega)^{-1} D_y. \end{aligned} \qquad (3)$$

Authorized licensed use limited to: University of Texas at Austin. Downloaded on April 10, 2009 at 10:06 from IEEE Xplore. Restrictions apply.

The field components are calculated in the successive way. For describing the permittivity and permeability, we used the lossy Drude model as given by

$$\varepsilon(\omega)=\varepsilon_{\circ}\left(1-\frac{\omega_{pe}^{2}}{\omega^{2}+j\omega\Gamma_{e}}\right),$$
$$\mu(\omega)=\mu_{\circ}\left(1-\frac{\omega_{pm}^{2}}{\omega^{2}+j\omega\Gamma_{m}}\right). \tag{4}$$

where ω_{pe} and ω_{pm} are the plasma frequencies of the electric and magnetic field and Γ_e and Γ_m are the loss factors, respectively.

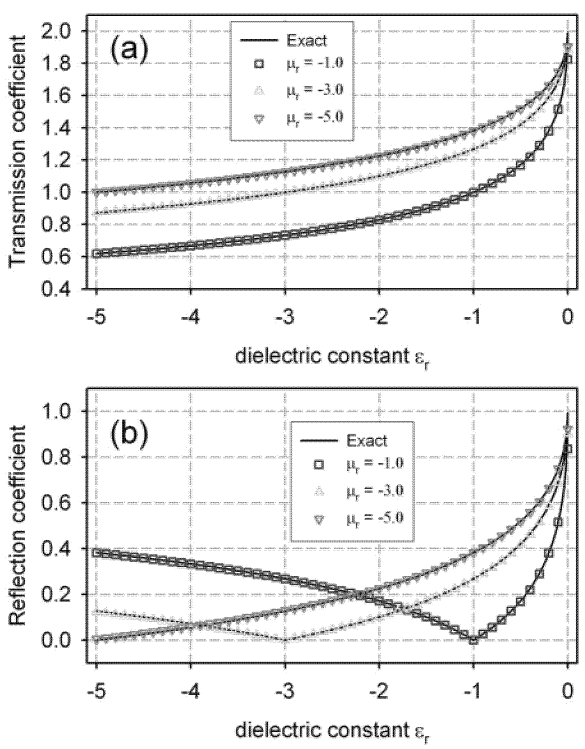

Fig. 1. The absolute value of (a) transmission coefficient and (b) reflection coefficient for different permeabilities as a function of relative permittivity in lossless DNG medium.

The monochromatic wave ($\omega_i = 2\pi f_i = 2\pi \times 1.0$ THz) is incident on the DNG material from free space at normal incidence. The wavelength of the incident wave in free space is $\lambda_{\circ} = c_{\circ} / f_i$ where $c_{\circ}$ is the speed of light in free space. The size of grid spacing is $\Delta z = \lambda_{\circ}/120$ and the time-step $\Delta t = \Delta z / c_{\circ}$ is used, satisfying the Courrant's stability condition [4]. The electric field in computation domain has 1001 Yee cells and the magnetic field node is located on the center of the grid spacing between the electric field node. The interface between free space and the DNG material is located on the center ($k_{center} = 501$) of the electric field grids and we used the average of the permittivities to either side as the permittivity value at this node. The end of computation domain meets the PML [11] slabs each having 200 nodes with a 4-power polynomial increase of distance divided by the total PML thickness and with the constant $\sigma_{\max} = 0.16/\Delta t$. The sinusoidal plane wave as an incident light source onto the DNG material is modeled using a total-field/scattered-field formulation [6]. In order to calculate the transmission and reflection coefficients, all the fields started being recorded after 5000 time step to obtain steady state. The transmitted field is recorded behind the DNG material interface and the reflected field is obtained by subtracting the incident field from the total field which is recorded in front of the interface. The recorded incident, transmitted and reflected fields are converted to the signals in frequency domain using a Fourier transform and then the transmission and reflection coefficients are obtained. Fig. 1 shows the examples of the transmission and reflection coefficient under various conditions of permittivity and permeability for lossless DNG material. It shows good agreement with the theoretical results.

CONCLUSION

We examined the transmission and reflection properties in DNG materials in one-dimensional FDTD formulation. The results are clearly in agreement with the theoretical results obtained from Maxwell's equations.

REFERENCES

[1] R. W. Ziolkowski and E. Heyman, "Wave propagation in media having negative permittivity and permeability," Phys. Rev. E, vol. 64, 056625 (2001).

[2] D. R. Smith et al., "Metamaterials and Negative Refractive Index," Science, vol. 305, 788-792 (2004).

[3] I. C. Khoo et al., "Nanosphere dispersed liquid crystals for tunable negative-zero-positive index of refraction in the optical and terahertz regimes," Opt. Lett., vol. 31, 2592-2594 (2006).

[4] A. Taflove and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain Method, 2nd Ed.*, Artech House, Inc., Norwood, MA, 2000.

[5] D. R. Smith and S. Schultz, "Determination of effective permittivity and permeability of metamaterials from reflection and transmission coefficients," Phys. Rev. B. vol. 65, 195104 (2002).

[6] R. W. Ziolkowski, "Design, Fabication, and Testing of Double Negative Metamaterials," IEEE Trans. Antennas Propagat., vol. 51, 1516-1529 (2003).

[7] T. M. Grzegorczyk et al., "Properties of Left-Handed MatamaterialsTransmission, Backward Phase, Negative Refraction, and Focusing," IEEE Trans. Microwave Theory Tech., vol. 53, 2956-2967 (2005).

[8] Vasundara Varadan et al., "Comparison of measurement and simulation of both amplitude and phase of reflected and transmitted fields in resonant Omega media," Microwave and Opt. Tech. Lett., vol. 48, 1549-155 (2006).

[9] R. Luebbers et al., A Frequency-Dependent Finite-Difference Time-Domain Formulation for Dispersive Materials, IEEE Trans. Electromagn. Compat., vol. 32, 222-227 (1990).

[10] K. S. Kunz and R. J. Luebbers, *Finite Difference Time Domain Method for Electromagnetics*, CRC Press LLC, Boca Raton, FL, 1993

[11] S. A. Cummer, "A simple, Nearly Perfectly Matched Layer for General Electromagnetic Media," IEEE Microwave Wireless Compon. Lett., vol. 13, 128-130 (2003).

Authorized licensed use limited to: University of Texas at Austin. Downloaded on April 10, 2009 at 10:06 from IEEE Xplore. Restrictions apply.