Measured Equation of Invariance Solution of 2-D Scattering Problems Using Line Sources as Metrons

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1 Introduction

The measured equation of invariance (MEI) was first introduced as a mesh truncation condition for the finite-difference (FD) or finite-element (FE) method in the analysis of open region electromagnetic scattering problems [1]. The method makes possible the truncation of the computation domain very close to the object surface while still maintaining the sparsity of the FD/FE matrix. However, it has been reported that the MEI method fails to give accurate results when the electrical size of the scatterer becomes large and a fixed number of nodes are coupled in the MEI [2]. Recently, *Xu* and *Chen* suggested that the accuracy of the MEI solutions could be improved by coupling more nodes in the MEI [3]. Unfortunately, adding more nodes in the MEI results in the aggravation of the ill-conditioning of the corresponding matrix used for the solution of MEI coefficients (called MEIC matrix) when sinusoidal functions are used as metrons. A consequence is that as the number of the coupled nodes increases, numerical integrations must be carried out with very high or even extremely high accuracy, which become impractically time-consuming and would completely destroy the efficiency of the original MEI method.

To overcome this difficulty, we choose δ functions as metrons to generate MEI coefficients. For two-dimensional (2-D) scattering problems, δ metrons are equivalent to line sources placed on the surface of the object. The most important advantage of δ metrons is that the corresponding measuring functions can be expressed analytically and no numerical integration is needed when computing the MEI coefficients. Considering that finding the MEI coefficients is the dominant part of the computation time consumed by the MEI method, we expect to greatly reduce the computation burden by using δ functions instead of sinusoidal functions as metrons. Besides, δ functions alleviate the ill-conditioning problem of the MEIC matrix. We have also found that the positioning of line sources must be investigated carefully in order to obtain convergent and accurate results. Our numerical practice shows that for geometry other than circular cylinders, uniformly distributed line sources do not always lead to convergent solutions as the number of coupled nodes increases. In this paper, we propose non-uniform positioning of δ metrons around the object surface to guarantee stable and accurate solutions. Numerical results show the efficiency of the technique, especially in handling 2-D electrically large scattering problems.

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2 Theory

The MEI method assumes that there exists a linear relation between the scattered field at a node on the mesh truncation boundary and those at its N neighbor nodes:

$$\sum_{i=0}^{N} a_i \Phi_i = 0 \tag{1}$$

where Φ_i (i=0, ..., N) is the scattered field at the i-th node. The MEI coefficients a_i 's are determined numerically by assuming certain surface current distributions (metrons) and evaluating the corresponding scattered fields (measuring functions) at the nodes. Sinusoidally distributed surface currents have been widely used as metrons, but they result in highly ill-conditioned matrix systems. This becomes even worse when one tries to increase the number of coupled nodes in the MEI equation, as suggested in [3]. As a consequence, numerical integrations must be carried out with very high accuracy for cylinders of arbitrary cross section, which could be extremely time-consuming. The difficulty can be circumvented by using δ functions as metrons. Varying the position of δ function on the object surface, we get M linearly independent metrons

$$J_m(\vec{r}\,') = \delta(\vec{r}\,' - \vec{r}_m), \qquad m = 1, \cdots, M \tag{2}$$

where $\vec{r}_m$'s are surface position vectors. Each of the metrons is equivalent to a line source placed on the surface of the object in 2-D cases. The scattered fields produced by these line sources can be obtained analytically without numerical integration:

$$\text{TM:} \qquad \Phi_i^m = E_{zi}^m = -\frac{\eta k}{4} H_0^{(2)}(k|\vec{r}_i - \vec{r}_m|) \tag{3}$$

$$\text{TE:} \qquad \Phi_i^m = H_{zi}^m = \frac{jk}{4} \cos\phi \, H_1^{(2)}(k|\vec{r}_i - \vec{r}_m|)\,. \tag{4}$$

In this way, we are able to establish the MEIC equations very quickly, which is especially favorable when the electrical size of the object becomes large. The use of δ metrons also greatly alleviates the ill-conditioning phenomenon of the MEIC equations when N becomes large. For example, the condition number is about 10^{21} for sinusoidal metrons and 10^{8} for δ metrons when N=9. Furthermore, the condition number increases much more slowly with increasing N for δ metrons than for sinusoidal metrons.

Despite the simplicity of the δ metron formulation, the positioning of the line sources demands careful considerations. Since we have found that uniformly distributed line sources fail to produce convergent results in some cases, non-uniform positioning of the line sources is proposed. In our numerical computation, the following non-uniform scheme is used:

$$L_m = \pm\left|\frac{2m}{M}\right|^{\alpha} \cdot \frac{P}{2}, \qquad m = 0,\ \pm 1,\ \dots,\ \pm\frac{M-1}{2} \tag{5}$$

where P is the perimeter of the cylinder, M is an odd number, and L_m is the perimetric length of a particular line source on the surface with respect to the line source nearest to the boundary node. The non-uniformity is controlled by the parameter α. Empirically, excellent solutions can be achieved by choosing α between 3 and 10. The distribution patterns for the uniform and non-uniform positioning are plotted in Fig. 1.

In principle, N metrons are enough to generate N MEI coefficients. Usually more metrons are used to guarantee more accurate and stable results. The resulting overspecified system is solved by regular least squares fitting.

3 Numerical Results

The relationship between the accuracy of the MEI solution and the number of coupled nodes N is shown in Fig. 2. The surface current density for a 50λ-diameter circular PEC cylinder in TE incidence is obtained with non-uniform positioning ($\alpha = 5.0$) and N=5, 15, and 25. The numerical solutions progressively approach the analytical solution when N increases. Although the increase of N will result in more nonzero elements in the FD-MEI matrix, they are still small portion of the total matrix elements. Thus, the sparsity of the FD-MEI matrix and the efficiency of the MEI method are still preserved.

For the case of a rectangular cylinder illuminated by a TE plane wave, the convergence behaviors of the numerical results as N increases with uniform and non-uniform positioning of line sources are shown in Fig. 3(a) and Fig. 3(b), respectively. It is evident that the uniform positioning does not lead to convergent result, while the non-uniform positioning produces stable and accurate solutions with increasing N.

Fig. 4 shows the surface current density around half perimeter of a $20\lambda \times 10\lambda$ rectangular PEC cylinder illuminated by a TM plane wave. Totally $2N$+1 line sources and non-uniform positioning ($\alpha = 5.0$) are used in the computation. The results provided by the MEI method when N equals to 21 are in very good agreement with the moment method solution.

References

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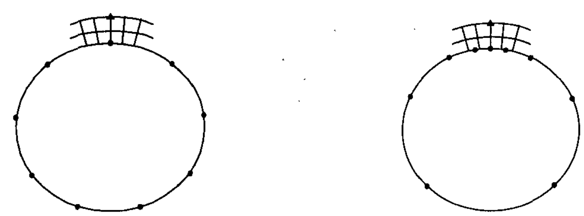

(a) (b)

Fig. 1 Distribution pattern for (a) uniform and (b) non-uniform positioning.

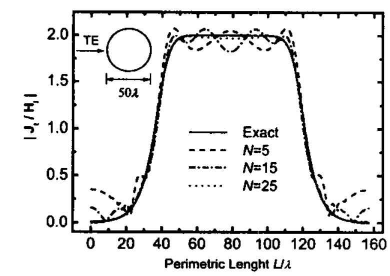

Fig. 2 Surface current density on a circular PEC cylinder of radius a=25λ, $\Delta\rho$=λ/20, $\Delta\theta$=$\Delta\rho/a$, α =5.0, TE incidence.

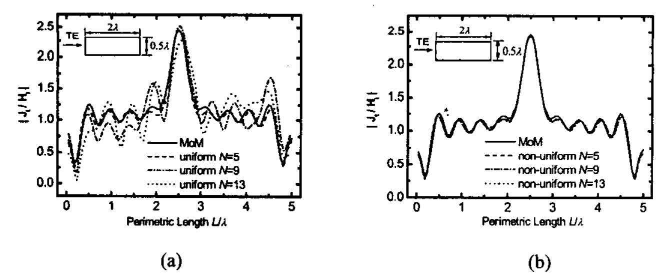

Fig. 3 Surface current density on a $2.0\lambda \times 0.5\lambda$ rectangular PEC cylinder, Δx=Δy=λ/20, TE incidence. (a) Uniform positioning. (b) Non-uniform positioning, α =5.0.

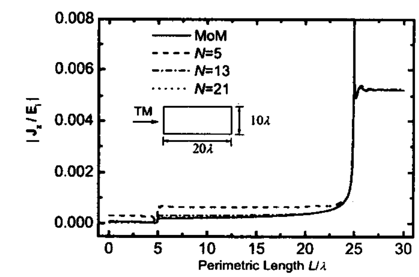

Fig. 4 Surface current density around half perimeter of a $20\lambda \times 10\lambda$ rectangular PEC cylinder, Δx=Δy=λ/20, α =5.0, TM incidence.