SCATTERING BY A 2D SQUARE SECTION METALLIC OBSTACLE

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ABSTRACT

The paper presents an application of the Wave Concept Iterative Process (WCIP) [1] in the case of the study of the scattering of an electromagnetic plane wave by a square section 2D obstacle. The 4 faces of the structure are studied independently, mutual influence being not taken into account. Current density results are presented and compared with those issued by the use of other scattering methods for several particular cases.

1. INTRODUCTION

Iterative methods are used more and more extensively in solving electromagnetic scattering problems in the last years, due to the augmentation of the computational capacities and to the simplicity of the formulation and the high efficiency of these methods. Generally speaking, it was shown [2] that the computational cost reduces from N^3 up to less than N, where N is the number of unknowns in the problem. Either stationary methods issued from the need of mathematical matrix manipulation, or issued from classic formulations [3], either non-stationary methods that have evolving formulation during the process [4], iterative methods are more and more often employed in the study of electromagnetic scattering problems.

However, the major disadvantages of the classic methods, mainly the restricted geometrical dimension range of applicability, remain unchanged for their iterative versions. As an example, we can mention the Iterative Physical Optics [5] that is limited to large scattering objects in terms of wavelength.

Among the most recent and the most efficient iterative methods, the **Wave Concept Iterative Process** was developed at first as an instrument for the study of in-guide and planar circuits scattering problems [1]. Due to its combined spectral – real domain formulation issued from the description of boundary conditions and modal behaviour and due to its applicability to all range of geometrical dimensions of the scaterer, the Wave Concept Iterative Process (WCIP) is highly recommended for its extension to free space electromagnetic diffraction studies. Already successfully tested in some simple classic free space scattering problems [6], the WCIP has a great potential of development to solving complex shaped structures and multiple obstacle electromagnetic problems [7].

The paper presents in the theory section a brief review of the principles of the WCIP, wave definitions and scattering operators. The WCIP is then applied in the case of a plane wave incidence upon a metallic 2D square section scattering obstacle. An approximation on the studied domain is introduced and the results are compared with those issued from the use of other field computation instruments, like the Moment Method [8], the MFIE [9], the Spectral Iteration Technique [10], and the Wavelet Expansion [11].

2. THEORY

2.1. The Wave Concept Iterative Process

The WCIP is an original method used for the study of scattering problems, either in-guide, either free space electromagnetic diffraction structures being solved.

The basic principle of the WCIP is the concept of waves, the incident one A, and the reflected one B, defined from the tangent electric field E, and the incident magnetic tangent field H. For simplicity reasons, instead of H, the current density vector J is preferred. The following relation defines the J vector:

$$\vec{J} = \vec{H} \times \hat{n} \tag{1}$$

where $\hat{n}$ is the normal versor to the scattering surface, defined in each point.

The waves A and B are defined by (2):

$$\begin{cases} \vec{A} = \dfrac{1}{2\sqrt{Z_0}}(\vec{E} + Z_0\vec{J}) \\ \vec{B} = \dfrac{1}{2\sqrt{Z_0}}(\vec{E} - Z_0\vec{J}) \end{cases} \tag{2}$$

where Z_0 is an arbitrary parameter.

The analytic expression of the iterative process is issued from the continuity and border conditions on the electromagnetic field. In the case of free space scattering, the iterative process is described by (3):

$$\begin{cases} \vec{B}_n + \vec{B}_0 = (I_s - I_m)(\vec{A}_{n-1} + \vec{A}_0) \\ \vec{A}_n = \hat{\Gamma}\vec{B}_n \end{cases} \tag{3}$$

where n is the number of the iteration, I_s and I_m are the magnetic and respectively metallic walls domain functions, $\hat{\Gamma}$ is the scattering operator, A_0 and B_0 are the waves corresponding to the incident electromagnetic field on the studied structure, defined as:

$$\begin{cases} \vec{A}_0 = \dfrac{1}{2\sqrt{Z_0}}(\vec{E}^i + Z_0\vec{J}^i) \\ \vec{B}_0 = \dfrac{1}{2\sqrt{Z_0}}(\vec{E}^i - Z_0\vec{J}^i) \end{cases} \tag{4}$$

The first equation of (3) is written in the real domain, the second one being written in the spectral domain, and therefore a modal transformation (FFT) is used to pass from one to the other.

The scattering operator is given by:

$$\hat{\Gamma} = \sum_m |f_m\rangle \frac{1 - Z_0 Y_m}{1 + Z_0 Y_m} \langle f_m| \tag{5}$$

where $\{f_m\}$ is a complete modal base and Y_m is the modal admittance of the m^{th} mode.

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2.2. Scattering by a 2D square section obstacle

Let us consider the incidence of an electromagnetic plane wave upon the 2D scatterer, like depicted in figure 1.

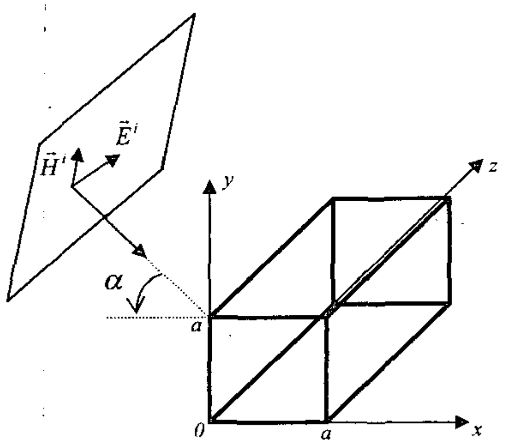

Figure 1. Wave incidence on the obstacle

The 4 faces of the studied structure are treated independently, but in a similar way. The square section and the numbering of the faces are depicted in figure 2.

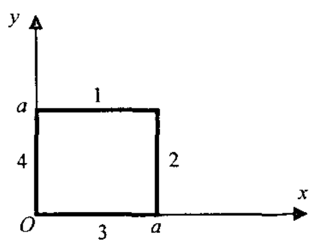

Figure 2. Square section of the obstacle

Let us consider the study domain of the face number 1. Theoretically, it extends from $x=-\infty$ to $x=+\infty$, for $y=a$. It would be numerically impossible to consider an infinity of points to study, and therefore we have to impose limitations by introducing an approximation. The domain will be therefore considered as periodic, with the period p, as suggested in figure 3.

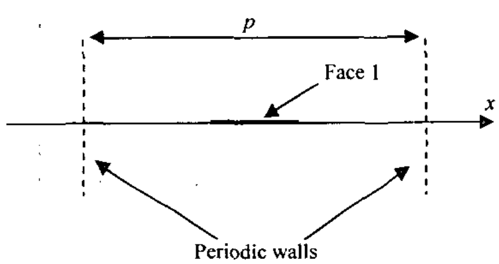

Figure 3. Approximation on the studied domain

The main steps used for the evaluation of the current density on each of the 4 faces are presented in the block diagram in figure 4.

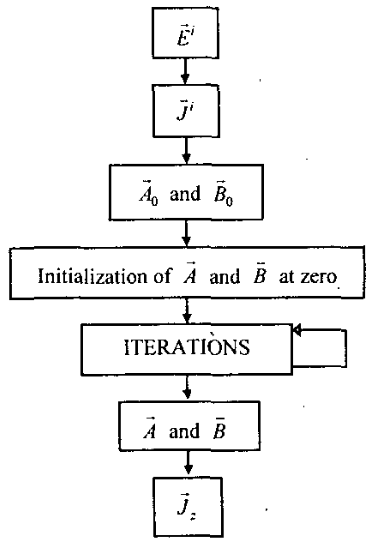

Figure 4. Block diagram of the current density calculation using WCIP

In the following, we are going to detail the steps of the block diagram from the figure above, for each of the 4 faces of the square section cylinder.

Let us consider (x_1, y_1, z_1) the rectangular coordinates system associated to the plane wave. The incident electric field has the expression:

$$\vec{E}^i = E_0 e^{-jkx_1}\vec{z}_1 \tag{6}$$

The incident electric field can be expressed in the main coordinates system by applying the following transformation:

$$\begin{cases} x_1 = x\cos\alpha - y\sin\alpha \\ y_1 = x\sin\alpha + y\cos\alpha \\ z_1 = z \end{cases} \tag{7}$$

We obtain:

$$\vec{E}^i = e^{-jk\cos\alpha x} e^{jk\sin\alpha y}\vec{z} \tag{8}$$

Using the following notations:

$$\begin{aligned} k_x &= k\cos\alpha \\ k_y &= k\sin\alpha \end{aligned} \tag{9}$$

the electric incident field is given by:

$$\vec{E}^i = e^{-jk_x x} e^{-jk_y y}\vec{z} \tag{10}$$

The correspondent current density is obtained by applying relation (1) for each face.

$$\vec{J}_0 = \frac{1}{\eta} e^{-jk_x x} e^{jk_y y}\left(\sin\alpha\,\vec{x} + \cos\alpha\,\vec{y}\right)\times\hat{n} \tag{11}$$

where η is the free space characteristic impedance (approx. 120π $[\Omega]$).

Let us now find the expressions of the initial waves A_0 and B_0 for each of the 4 faces.

Face1. The first face of the square section is characterised by:

$$\begin{cases} y = a \\ \hat{n} = \vec{y} \end{cases} \tag{12}$$

The correspondent expressions for the incident wave fields are given by:

$$\begin{cases} E_1^i = e^{-jk_x x} e^{jk_y a} \\ J_{01} = \dfrac{\sin\alpha}{\eta} e^{-jk_x x} e^{jk_y a} \end{cases} \tag{13}$$

Therefore, we obtain for the initial waves the following expressions:

$$\begin{cases} A_0^1 = \left(\dfrac{1}{2\sqrt{Z_0}} + \dfrac{\sin\alpha\sqrt{Z_0}}{2\eta} \right) e^{-jk_x x} e^{jk_y a} \\ B_0^1 = \left(\dfrac{1}{2\sqrt{Z_0}} - \dfrac{\sin\alpha\sqrt{Z_0}}{2\eta} \right) e^{-jk_x x} e^{jk_y a} \end{cases} \tag{14}$$

Face2. The second face of the square section is characterised by:

$$\begin{cases} x = a \\ \vec{n} = \vec{x} \end{cases} \tag{15}$$

The correspondent expressions for the incident wave fields are given by:

$$\begin{cases} E_2^i = e^{-jk_x a} e^{jk_y y} \\ J_{02} = -\dfrac{\cos\alpha}{\eta} e^{-jk_x a} e^{jk_y x} \end{cases} \tag{16}$$

Therefore, we obtain for the initial waves the following expressions:

$$\begin{cases} A_0^2 = \left(\dfrac{1}{2\sqrt{Z_0}} - \dfrac{\cos\alpha\sqrt{Z_0}}{2\eta} \right) e^{-jk_x a} e^{jk_y y} \\ B_0^2 = \left(\dfrac{1}{2\sqrt{Z_0}} + \dfrac{\cos\alpha\sqrt{Z_0}}{2\eta} \right) e^{-jk_x a} e^{jk_y y} \end{cases} \tag{17}$$

Face3. The third face of the square section is characterised by:

$$\begin{cases} y = 0 \\ \vec{n} = -\vec{y} \end{cases} \tag{18}$$

The correspondent expressions for the incident wave fields are:

$$\begin{cases} E_3^i = e^{-jk_x x} \\ J_{03} = -\dfrac{\sin\alpha}{\eta} e^{-jk_x x} \end{cases} \tag{19}$$

Therefore, we obtain for the initial waves the expressions:

$$\begin{cases} A_0^3 = \left(\dfrac{1}{2\sqrt{Z_0}} - \dfrac{\sin\alpha\sqrt{Z_0}}{2\eta} \right) e^{-jk_x x} \\ B_0^3 = \left(\dfrac{1}{2\sqrt{Z_0}} + \dfrac{\sin\alpha\sqrt{Z_0}}{2\eta} \right) e^{-jk_x x} \end{cases} \tag{20}$$

Face4. The 4-th face of the square section is characterised by:

$$\begin{cases} x = 0 \\ \vec{n} = -\vec{x} \end{cases} \tag{21}$$

The correspondent expressions for the incident wave fields are given by:

$$\begin{cases} E_4^i = e^{jk_y y} \\ J_{04} = \dfrac{\cos\alpha}{\eta} e^{jk_y x} \end{cases} \tag{22}$$

Therefore, we obtain for the initial waves the expressions:

$$\begin{cases} A_0^4 = \left(\dfrac{1}{2\sqrt{Z_0}} + \dfrac{\cos\alpha\sqrt{Z_0}}{2\eta} \right) e^{jk_y y} \\ B_0^4 = \left(\dfrac{1}{2\sqrt{Z_0}} - \dfrac{\cos\alpha\sqrt{Z_0}}{2\eta} \right) e^{jk_y y} \end{cases} \tag{23}$$

Taking into account the way of treating independently each face, we need to introduce two types of modal bases, one depending of x, the other one having as variable y. The expressions of the functions of the modal bases are given by:

$$f_{mx} = \frac{1}{\sqrt{p}} e^{j\frac{2\pi}{p}mx} \tag{24}$$

and, respectively:

$$f_{my} = \frac{1}{\sqrt{p}} e^{j\frac{2\pi}{p}my} \tag{25}$$

where p is the period of the walls (figure 3) and the coefficient $\frac{1}{\sqrt{p}}$ comes from the orthonormation of the bases.

The correspondent modal impedance of the m-th mode is given by:

$$Z_m = \frac{1}{j\omega\varepsilon}\sqrt{\left(\frac{2\pi m}{p}\right)^2 - k^2} \tag{26}$$

We have now the expressions of all the necessary terms and the iterative process can be initiated.

3. NUMERICAL RESULTS

We have applied WCIP to several particular cases of diffraction of a plane wave by a square section metallic 2D structure and we have compared our results with those issued from the use of other field computation methods.

Figure 5 presents the case $a = \lambda$ and $\alpha = 45°$. The Magnetic Field Integral Equation (MFIE) [9] is used to compare the two sets of results.

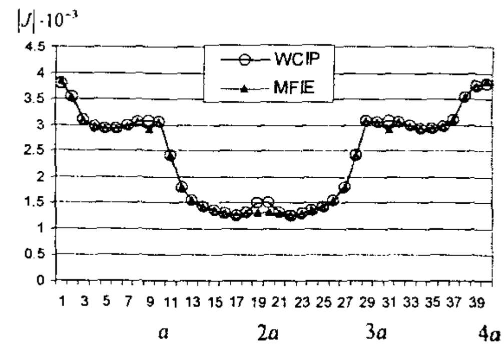

Figure 5. Current density for $a = \lambda$ and $\alpha = 45°$. Comparison with MFIE [9]

The current density is traced all around the square section of the obstacle, as suggested bellow:

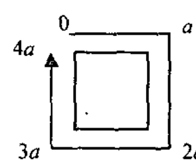

Figure 6. Development of the square section for the presentation of the results

The following studied particular case is $a = \lambda$ and $\alpha = 90°$. The results are compared with the Moment Method [8].

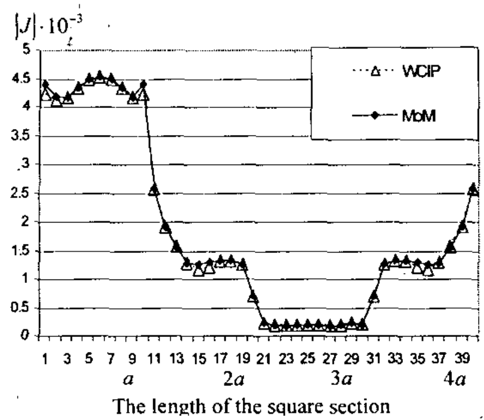

Figure 7. Current density for $a = \lambda$ and $\alpha = 90°$. Comparison with The MoM [8]

The Spectral Iterative Technique [10] is used to validate the results given by WCIP for the case $a = 3\lambda$ and $\alpha = 45°$.

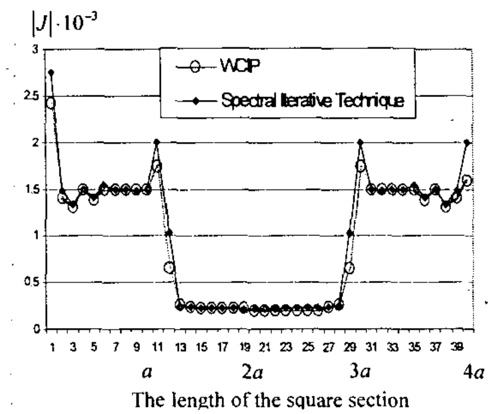

Figure 8. Current density for $a = 3\lambda$ and $\alpha = 45°$. Comparison with the Spectral Iterative Technique [10]

The last particular case we present is a comparison between WCIP and the Wavelet Expansion Method [11], for $a = 1.5\lambda$ and $\alpha = 0°$.

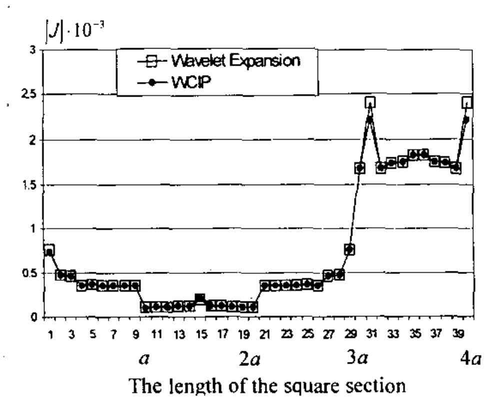

Figure 9. Current density for $a = 1.5\lambda$ and $\alpha = 0°$. Comparison with the Wavelet Expansion Method [11]

One can notice an excellent matching between the results given by the Wave Concept Iterative Process and those issued from the use of other field computation methods.

4. CONCLUSIONS

An application of the WCIP was realised in the case of the study of the scattering of an electromagnetic plane wave by a 2D square section metallic obstacle. The principles of the Wave Concept Iterative Process were reminded, and several particular cases of incidence and geometrical dimensions were treated. The current densities were compared with the results given by other methods, being in very good accord. In conclusion, WCIP was successfully validated for this type of scattering obstacles. The perspectives of our work include taking into account the influence of the radiation of one face over the other faces, and taking into study more complex section metallic obstacles.

5. REFERENCES

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